Characteristic Polynomials of one Side Weighted Adjacency Matrices of Linear Chains

MILAN KUNZ

Abstract

Characteristic polynomials of one side arithmetically weighted adjacency matrices of linear chains were calculated. The elements of the inverse of their matrix are derived from odd factorials.

Key words: adjacency matrices, Cluj matrices, eigenvalues, characteristic polynomials.

Diudea [1,2] introduced asymmetrically weighted distance matrices, Cluj matrices, by the Wiener weights $N_{i,(i,j)}$ and $N_{j,(i,j)}$ (the number of vertices on the end j of the path p_{ij} from the diagonal vertex $(i = j)$ to the off-diagonal vertex j $(i \neq j)$. I have studied [3] some properties of the direct (Hadamard) product of a Cluj matrix with the corresponding adjacency matrix $\mathbf{A}$:

$$\mathbf{C}_e = \mathbf{C}_p \bullet \mathbf{A} \tag{1}$$

which leaves only adjacent elements of the Cluj matrix $\mathbf{C}_e$ (or equivalently Cluj weighted adjacency matrix $\mathbf{A}_C$, for example for the linear chain L_4 (n-butane)

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The eigenvalues of the linear chains L_n with odd n (from the inspection of the first chains) have values 0, $[2,\ 4,\ \ldots (n-1)]$, the eigenvalues of the linear chains L_n with even n have values $[1,\ 3,\ \ldots (n-1)]$.

In this paper, the characteristic polynomials of one side arithmetically weighted adjacency matrices of linear chains are studied

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{pmatrix}$$

The characteristic polynomials were calculated by counting weighted k-tuples. The results are tabulated:

Table 1: Coefficients of weighted linear chains adjacency matrices

n				
1	1			
2	1	-1		
3	1	-3		
4	1	-6	3	
5	1	-10	15	
6	1	-15	45	-15
7	1	-21	105	-105

The coefficients of the table are $t_{i,1} = 1$, $t_{i,j} = (n - j + 1)t_{i-1,j-1} + t_{i-1,j}$. These coefficients can be tabulated in following matrix according to the powers of x terms

1	0	0	0	0	0	0
-1	1	0	0	0	0	0
0	-3	1	0	0	0	0
0	3	-6	1	0	0	0
0	0	15	-10	1	0	0
0	0	-15	45	-15	1	0
0	0	0	-105	105	-21	1

The inverse of this matrix is

1	2	3	4	5	6	7	$\sum$
1	0	0	0	0	0	0	1
1	1	0	0	0	0	0	2
3	3	1	0	0	0	0	5
15	15	6	1	0	0	0	37
105	105	45	10	1	0	0	266
945	945	420	105	15	1	0	2431
10395	10395	0	105	105	21	1	27007

The elements of the first column are the odd factorials $1x1x3x5x7\dots$ (the first 1 is 0!).

The recurrence of the matrix elements is $m_{1,1} = 1$, otherwise

$$[2(n-1) - j]m_{i-1,j} + m_{i-1,j-1} \tag{2}$$

The row sums S, except the first two, are obtained as

$$(2n-1)S_{n-1} + S_{n-2} \tag{3}$$

The characteristic polynomials of odd and even chains differ. It were better to include empty side diagonals and the epty graph. The reccurence is then:

1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
-1	0	1	0	0	0	0	0
0	-3	0	1	0	0	0	0
3	0	-6	0	1	0	0	0
0	15	0	-10	0	1	0	0
-15	0	45	0	-15	0	1	0
0	-105	0	105	0	-21	0	1

The inverse of this matrix has the same elements but they are all positive:

1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
1	0	1	0	0	0	0	0
0	3	0	1	0	0	0	0
3	0	6	0	1	0	0	0
0	15	0	10	0	1	0	0
15	0	45	0	15	0	1	0
0	105	0	105	0	21	0	1

The recurrence of the matrix elements is $m_{1,1} = 1$, otherwise

$$(n-1]m_{i-2,j} + m_{i-1,j-1} \tag{4}$$

The row sums S, except the first two, are obtained as

$$(n-1)S_{n-2} + S_{n-1} \tag{5}$$

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